Analysis of Calving Events in Antarctic Ice Shelves Using Configurational Forces

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Previous studies on the sensitivity of cracks in ice shelves with different boundary conditions, stress states and density profiles revealed the need for further analyses. As the transfer of boundary conditions from dynamic ice flow simulations to the linear elastic fracture analyses proved to be a critical point in previous studies, a new approach to relate viscous and elastic material behaviour is proposed. The numerical simulations are conducted using Finite Elements utilizing the concept of configurational forces. To show the applicability of the approach, a 2-dimensional plane stress geometry with volume loads due to the ice shelf flow is analyzed. The resulting crack path is compared to available crack paths from satellite images.

1 From viscous flow to elastic fracture

1.1 Viscous volume forces

In a first approach, ice can be regarded as a visco-elastic material represented by a Maxwell element that consists of a linear elastic spring and a nonlinear dashpot. For this material the stress can be written as $\boldsymbol{\sigma}_{\mathrm{v}} = \boldsymbol{\sigma}_{\mathrm{e}} = \boldsymbol{\sigma}$ with the elastic stress $\boldsymbol{\sigma}_{\mathrm{e}}$ and the viscous stress $\boldsymbol{\sigma}_{\mathrm{v}}$. Under steady state conditions and the absence of volume forces, the equilibrium condition yields

$$\operatorname{div}\boldsymbol{\sigma} = \operatorname{div}\boldsymbol{\sigma}_{\mathrm{e}} - \operatorname{div}\boldsymbol{\sigma}_{\mathrm{v}} = \mathbf{0}. \tag{1}$$

For a given horizontal ice flow field with the velocities $\boldsymbol{v}(x, y)$ in the plane of the shelf, the viscous surface stress $\boldsymbol{\sigma}_{\mathrm{v}}$ can be evaluated using Glen's flow law [1] yielding, $\boldsymbol{\sigma}^{\mathrm{v}} = Bd^{\frac{1-n}{n}}\,(\mathrm{tr}\boldsymbol{D}\mathbf{1} + \boldsymbol{D})$, with the effective strain rate $d = \sqrt{\frac{1}{2}\,\mathrm{tr}(\boldsymbol{D}^2)}$, the strain rate tensor $\boldsymbol{D}$ and the material parameters $B \approx 7 \cdot 10^7 \mathrm{Pa\,s}^{\frac{1}{3}}$ and $n = 3$. If we now understand the divergence of the viscous surface stress as a negative viscous volume force $\boldsymbol{f}_{\mathrm{v}} = -\operatorname{div}\boldsymbol{\sigma}^{\mathrm{v}}$, Eqn. (1) can be rewritten as

$$\operatorname{div}\boldsymbol{\sigma}_{\mathrm{e}} + \boldsymbol{f}_{\mathrm{v}} = \mathbf{0}. \tag{2}$$

Instead of using stress or equivalent displacement boundary conditions, which are difficult to determine, the ice shelf is loaded by viscous volume forces yielding a stress field in the unfractured ice shelf that represents the viscous stresses.

1.2 Fracture mechanical analysis

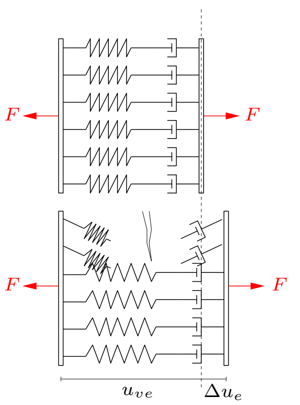

Fig. 1: Cracking in a Maxwell material (schematic)

An accurate spatial representation of the stress field yields the possibility to study the fracture path of given initial cracks emerging from the boundaries through the shelf. For this purpose, we assume that during the short period of the fracture process, only the elastic response has to be considered and strains due to the long term flow behaviour are negligible small (Fig. 1). The criticality of the stress state at the crack tip is evaluated using configurational forces. The configurational balance of linear momentum

$$\operatorname{div}\boldsymbol{\Sigma} + \boldsymbol{g} = \mathbf{0}, \tag{3}$$

with the Eshelby stress tensor $\boldsymbol{\Sigma} = W\mathbf{1} - (\nabla\boldsymbol{u})^T\boldsymbol{\sigma}_{\mathrm{e}}$ and the configurational volume force can be rearranged to yield

$$\boldsymbol{g} = -(\nabla\boldsymbol{u})^T\boldsymbol{f}_{\mathrm{v}} - \left.\frac{\partial W}{\partial\boldsymbol{x}}\right|_{\mathrm{expl.}} \rightarrow \boldsymbol{G} = -\left.\frac{\partial W}{\partial\boldsymbol{x}}\right|_{\mathrm{expl.}} = -\operatorname{div}\left[W\mathbf{1} - (\nabla\boldsymbol{u})^T\boldsymbol{\sigma}_{\mathrm{e}}\right] + (\nabla\boldsymbol{u})^T\boldsymbol{f}_{\mathrm{v}}. \tag{4}$$

In this equation W is the elastic strain energy density $W = \frac{1}{2}\boldsymbol{\varepsilon} : (\mathbf{C}\boldsymbol{\varepsilon})$. For details on configurational forces and their numerical implementation the reader is referred to e.g. [2]. The crack propagates in the direction of the crack driving force $\boldsymbol{G}$ by a constant increment Δa if the absolute value of $\boldsymbol{G}$ exceeds a critical value G_{c}.

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2 Results: Application to Pine Island Glacier

Pine Island Glacier, located at the Western Antarctic Peninsula, is the longest and fastest floating glacier of western Antarctica. On a decadal time scale, horizontal fractures propagate through the ice shelf and ice bergs with an area of several hundreds of square kilometres calve. This process happens again since October 2011 and is monitored using radar remote sensing products. The availability of remote sensing surface displacement data enables us to validate the fracture mechanical model.

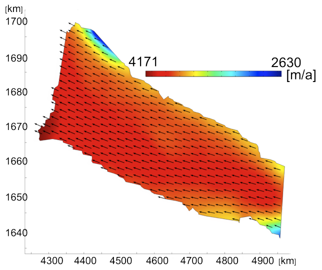

Fig. 2: Abs. flow velocities [3]

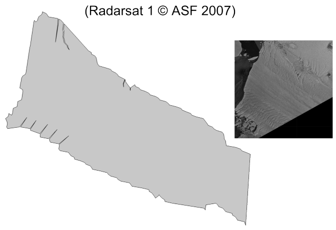

Fig. 3: Initial crack geometry

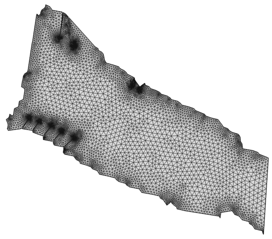

Fig. 4: Finite element mesh

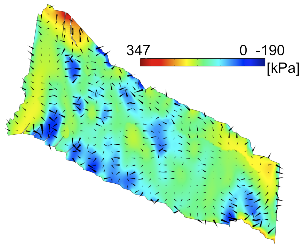

Fig. 5: Viscous surface stress and resulting volume force $\boldsymbol{f}_{\mathrm{v}}$

Figure 2 shows the absolute velocities and the flow direction of an average velocity field resulting from satellite pictures of the years 2006 and 2008. In Fig. 3, the initial cracks based on a satellite image from 2007 are indicated. The finite element mesh produced by the FE program COMSOL with mesh refinement at every crack tip is shown in Fig. 4. The refinement is performed by dummy nodes in 50m distance from the respective crack tip, resulting in an maximum element length at the crack tip of 50m. The mesh consists of plane quadratic triangles with 3 internal Gauß points. For every crack increment, a new mesh with about 12500 elements and 25000 nodes is generated. Figure 5 shows the viscous surface stresses and the direction of the resulting viscous volume forces.

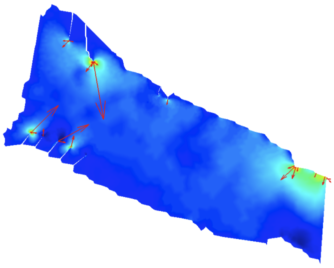

Fig. 6: Configurational forces at the crack tip

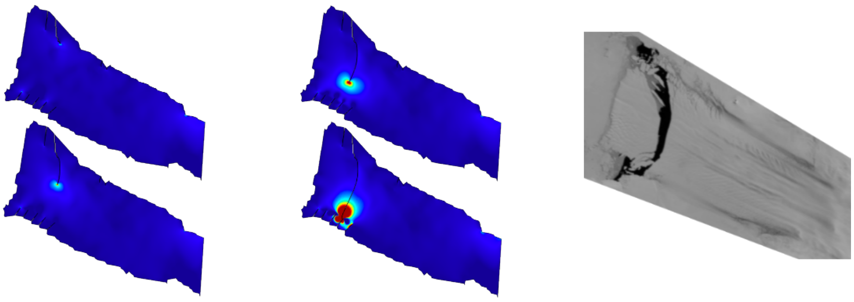

Fig. 7: Evolving crack path and final crack 2007 (MODIS)

The principal elastic stresses and the resulting configurational forces at the crack tip are shown in Fig. 6. The evolving crack path in comparison to the calving event of 2007 is depicted in Fig. 7. The resulting crack path is in good agreement with the actual path of 2007.

3 Conclusion

The approach of transforming viscous stresses into volume forces for a short term linear elastic fracture analysis provides good results in comparison to actual fracture pattern without the costs of a full visco-elastic simulation. However, for a validation reasons, more data of calving events with the previous velocity fields are desirable.

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