PAMM · Proc. Appl. Math. Mech. **11**, 169 – 170 (2011) / **DOI** 10.1002/pamm.201110076

Fracture Mechanical Analysis of Cracks in Ice Shelves using the Finite Element Method and Configurational Forces

Carolin Plate[1,*], **Ralf Müller**[1], **Dietmar Gross**[2], and **Angelika Humbert**[3]

[1] Institute of Applied Mechanics, Department of Mechanical and Process Engineering, TU Kaiserslautern
[2] Division of Solid Mechanics, Department of Civil Engineering & Geodesy, TU Darmstadt
[3] Institute of Geophysics, KlimaCampus, University of Hamburg

Ice shelves are important elements of the climate system and sensitive to climate changes. The disintegration of large Antarctic ice shelves is the focus of this fracture mechanical analysis. Ice is a complex material which, depending on the context, can be seen as a viscous fluid or as an elastic solid. A fracture event usually occurs on a rather short time scale, thus the elastic response is important and linear elastic fracture mechanics can be used. The investigation of the stress intensity factor as a measure of crack tip loading is based on a 2-dimensional analysis of a single crack with a mode-I type load and additional body loads. This investigation is performed using configurational forces. Depth dependent density and temperature profiles are considered. The relevant parameters are obtained by literature, remote sensing data analysis and modeling of the ice dynamics. The criticality of wet surface cracks is investigated.

1 Energy release rate and configurational forces

The analysis of the criticality of cracks in ice shelves is based on the evaluation of the stress intensity factor (SIF) at the crack tip using configurational forces. The configurational balance of linear momentum

$$\operatorname{div} \boldsymbol{\Sigma} + \boldsymbol{g} = \boldsymbol{0}, \tag{1}$$

with the Eshelby stress tensor $\boldsymbol{\Sigma} = U\mathbf{1} - (\nabla \boldsymbol{u})^T \boldsymbol{\sigma}$ and the configurational volume force can be rearranged to yield

$$\boldsymbol{g} = -(\nabla \boldsymbol{u})^T \boldsymbol{f} - \left.\frac{\partial U}{\partial \boldsymbol{x}}\right|_{\text{expl.}} \rightarrow - \left.\frac{\partial U}{\partial \boldsymbol{x}}\right|_{\text{expl.}} = -\operatorname{div}\left[U\mathbf{1} - (\nabla \boldsymbol{u})^T \boldsymbol{\sigma}\right] + (\nabla \boldsymbol{u})^T \boldsymbol{f}. \tag{2}$$

In this equation $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\boldsymbol{u}$ the actual displacement, $\boldsymbol{f}$ is the physical volume force and U is the strain energy density $U = \frac{1}{2}\boldsymbol{\varepsilon} : (\mathbf{C}\boldsymbol{\varepsilon})$. Discretising $(2)_2$ with finite elements reveals the driving force at the crack tip G. For details the reader is referred to e.g. [3]. From the crack driving force the stress intensity factor K_I can be calculated using the relation

$$K_1 = \sqrt{G \frac{E}{1-\nu^2}} \tag{3}$$

for the plane strain case.

2 Model of a floating ice shelf

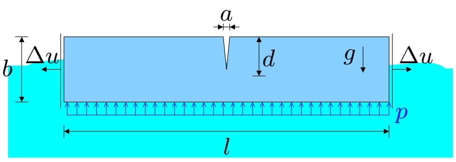

Fig. 1 Plane strain model

Fig. 1 illustrates the plane strain model, representing a vertical cut through an 'infinite' ice shelf. The model takes different displacement boundary conditions, gravity, different density profiles as well as water load on the crack faces into account. The displacement boundary conditions are derived from surface stresses σ, using the relation

$$\Delta u = \frac{\sigma(1-\nu^2)}{E}\frac{l}{2}, \tag{4}$$

with a Young's modulus of $E = 5 \cdot 10^9$ kPa and a Poisson's ratio of $\nu = 0.3$. The length of the ice shelf model is 2000m and the thickness 250m. The stresses used are based on typical velocities measured in the Wilkins Ice Shelf [1] and range from 0 to 300 kPa.

* Corresponding author E-mail: plate@rhrk.uni-kl.de, Phone: +49 631 205 2127, Fax: +49 631 205 2128

3 Results

The solution of the finite element problem is done by COMSOL, with an additional code in MATLAB is required for the evaluation of the stress intensity factors. All simulations are conducted using 6-node triangular elements with quadratic shape functions.

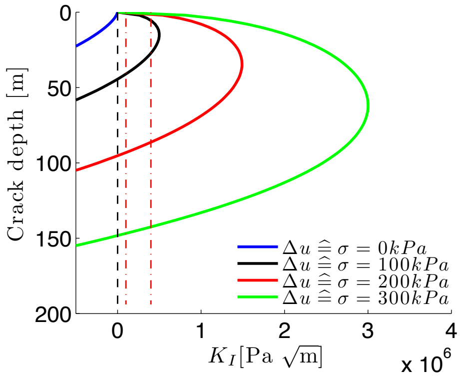

Fig. 2 K_I for different tensile loads

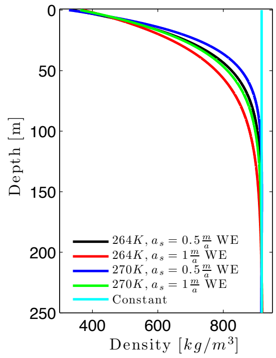

Fig. 3 Density profiles

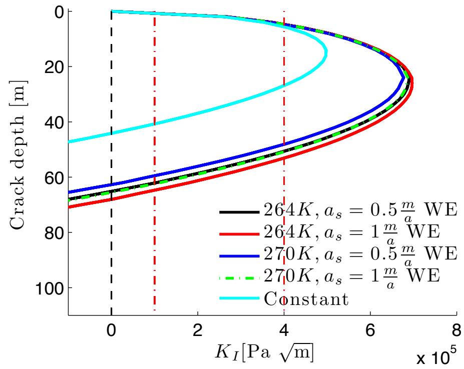

Fig. 4 K_I for different density profiles

Fig. 2 shows the SIF K_I for different displacement boundary conditions. It is obvious, that for zero boundary displacement, only the overburden pressure of the ice is acting on the crack. The negative SIF can be interpreted as crack closure. Larger tensile loads lead to positive stress intensity factors for shallow cracks. For deeper cracks the pressure of the ice becomes predominant and leads to closure. Critical SIF are indicated with the red dashed lines and range from 1 to $4{\cdot}10^5 Pa\sqrt{m}$.

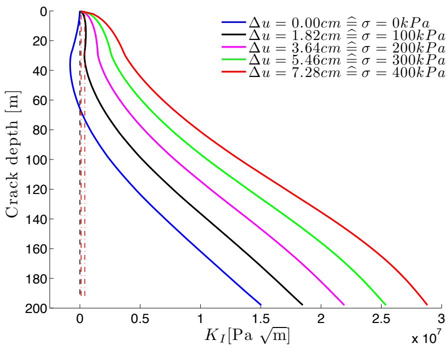

Fig. 5 Water filled cracks

Fig. 3 presents exponential fits for density profiles estimated from a densification model of [2] as well as a constant profile. The SIF in Fig. 4 show, that the small differences in the density profiles lead to only marginal differences in the K_I, strongly varying profiles lead to lower K_I than more moderate profiles. The constant profile leads to a higher overburden pressure and therefore to a lower K_I. The SIF for cracks subjected to water pressure acting on the crack faces are shown in Fig. 5. The cracks are filled up to sea level leading to unfilled shallow cracks with equal SIF as presented in Fig.2. Cracks deeper than sea level show strongly increasing stress intensity factors that indicate a break through of deep cracks even for zero tensile loading. Further simulations with varying Poisson's ratio show a strong influence of the Poisson's ratio on the SFI. This can be explained by the transformation of the vertically acting ice overburden pressure into horizontal stresses.

4 Conclusion

Unfilled cracks do not reach the bottom of the simulated ice shelf, under the given loading, whereas little water is sufficient for a break through of cracks. Varying density profiles and different Poisson's ratios change the stress intensity factors. More research has to be done on depth dependent material properties and more realistic crack geometries (3D).

Acknowledgements This study was supported by the German Research Foundation (DFG) under grants MU 1370 14-1 and the cluster of excellence CliSAP at the Klima Campus of the University of Hamburg.

References

[1] M. Braun, A. Humbert, A. Moll, Changes of the Wilkins Ice Shelf over the past 15 years and inferences on its stability, The Cryosphere Discuss., **2**, 341-382 (2009)
[2] M. M. Herron, Ch. C. Langway, Firn densification: an empirical Model, J. Glaciol., **25**, 373-885 (1980)
[3] R. Müller, S. Kolling, D. Gross, On configurational forces in the context of the Finite Element Method, Int. J. for Numerical Methods in Engineering, **53**, 1557-1574 (2002)