

CONTENTS

Preface	vii
Preface to the Second Edition	xiii
Introduction	1
Onset of Turbulence	3
Part One -- Classical Concepts in Turbulence Modeling	10
Chapter I. Turbulent Flow	10
1. Equations of Fluid Dynamics and Their Consequences	10
1.1 Reynolds' Averaging Technique	10
1.2 Equations of Fluid Dynamics	12
1.3 Equation of Kinetic Energy	15
1.4 Equation of Heat Conduction	18
2. Reynolds' Stresses	21
2.1 Physical and Geometrical Interpretation of Reynolds' Stresses	21
2.2 Eddies and Eddy Viscosity	23
2.3 Poiseuille and Couette Flow	28
3. Length Theory	40
3.1 Prandtl's Mixing Length Theory	40
3.2 Mixing Length in Taylor's Sense	45
3.3 Betz's Interpretation of von Kármán's Similarity Hypothesis	48
4. Universal Velocity Distribution Law	51
4.1 Prandtl's Approach	51
4.2 von Kármán's Approach	53
4.3 Turbulent Pipe Flow with Porous Wall	53
5. The Turbulent Boundary Layer	59
5.1 Turbulent Flow Over a Solid Surface	59
5.2 Law of the Wall in Turbulent Channel Flow	63
5.3 Velocity Distribution in Transient Region of a Moving Viscous Turbulent Flow	71
5.4 A New Approach to the Turbulent Boundary Layer Theory Using Lumley's Extremum Principle	80

Part Two -- Statistical Theories in Turbulence	92
Chapter II. Fundamental Concepts	92
6. Stochastic Processes	92
6.1 General Remarks	92
6.2 Fundamental Concepts in Probability	93
6.3 Random Variables and Stochastic Processes	95
6.4 Weakly Stationary Processes	112
6.5 A Simple Formulation of the Covariance and Variance for Incompressible Flow	124
6.6 The Correlation and Spectral Tensors in Turbulence	128
6.7 Theory of Invariants	138
6.8 The Correlation of Derivatives of the Velocity Components	141
7. Propagation of Correlations in Isotropic Incompressible Turbulent Flow	145
7.1 Equations of Motion	145
7.2 Vorticity Correlation and Vorticity Spectrum	148
7.3 Energy Spectrum Function	151
7.4 Three-Dimensional Spectrum Function	157
Chapter III. Basic Theories	161
8. Kolmogoroff's Theories of Locally Isotropic Turbulence	161
8.1 Local Homogeneity and Local Isotropy	161
8.2 The First and the Second Moments of Quantities $w_i(x_j)$	164
8.3 Hypotheses of Similarity	169
8.4 Propagation of Correlations in Locally Isotropic Flow	173
8.5 Remarks Concerning Kolmogoroff's Theory	176
9. Heisenberg's Theory of Turbulence	178
9.1 The Dynamical Equation for the Energy Spectrum	178
9.2 Heisenberg's Mechanism of Energy Transfer	181
9.3 von Weiszäcker's Form of the Spectrum	192
9.4 Objections to Heisenberg's Theory	193
10. Kraichnan's Theory of Turbulence	196
10.1 Burgers' Equation in Frequency Space	197
10.2 The Impulse Response Function	206
10.3 The Direct Interaction Approximation	209
10.4 Third Order Moments	213
10.5 Determination of Green's Function	217
10.6 Summary of Results of Burgers' Equation in Kraichnan's Sense	220
11. Application of Kraichnan's Method to Turbulent Flow	221
11.1 Derivation of Navier-Stokes Equation in Fourier Space	221
11.2 Impulse Response Function for Full Turbulent Representation	225
11.3 Formal Statement by Direct-Interaction Procedure	227
11.4 Application of the Direct-Interaction Approximation	228
11.5 Averaged Green's Function for the Navier-Stokes Equations	230

12. Hopf's Theory of Turbulence	232
12.1 Formulation of the Problem in Phase Space and the Characteristic Functional	233
12.2 The Functional Differential Equation for Phase Motion	238
12.3 Derivation of the ϕ -Equation	243
12.4 Elimination of Pressure Functional π from the ϕ -Equation	245
12.5 Forms of the Correlation for $n=1$ and $n=2$	246
Chapter IV. Magnetohydrodynamic Turbulence	251
13. Magnetohydrodynamic Turbulence by Means of a Characteristic Functional	251
13.1 Formulation of the Problem in Phase Space	253
13.2 ϕ -Equations in Magnetohydrodynamic Turbulence . .	259
13.3 Correlation Equations	263
14. Wave-Number Space	269
14.1 Transformation to Wave-Number Space	269
14.2 The Spectrum Equations and Additional Conservation Laws	279
14.3 Special Case of Isotropic Magnetohydrodynamic Turbulence	286
15. Stationary Solution for ϕ-Equations	291
15.1 Stationary Solution for the Case $\lambda=v=0$	291
15.2 Solution to the ϕ -Equations for Final Stages of Decay	296
16. Energy Spectrum	301
16.1 Energy Spectrum in the Equilibrium Range	301
16.2 Extension of Heisenberg's Theory in Magnetohydrodynamic Turbulence	303
17. Temperature Dispersion in Magnetohydrodynamic Turbulence	309
17.1 Turbulent Dispersion	309
17.2 Formulation of the Problem	312
17.3 Universal Equilibrium	316
18. Temperature Spectrum for Small and Large Joule Heat Eddies	322
18.1 Small Joule Heat Eddies	322
18.2 Large Joule Heat Eddies	353
19. The Temperature Spectrum for the Joule Heat Eddies of Various Sizes	354
19.1 The Viscous Dissipation Process	354
19.2 The Joule Heat Model	356
19.3 The Calculation of the Temperature Spectrum . .	360
19.4 Effect of Viscous Dissipation on the Temperature Distribution	369
20. Thomas' Numerical Experiments	372
20.1 Turbulent Dynamo Competing Processes	372
20.2 Nondissipative Model System $\lambda=v=0$	374
20.3 Numerical Experiments	376

21. Some Further Improvements of Dispersion Theory in Magnetohydrodynamic Turbulence	383
21.1 Remarks on the Turbulent Dispersion of Temperature for $R_m \gg R \gg 1$	383
21.2 Heat Equation for Conductive Cut-Off Wave Number for $H(k)$	385
21.3 Solution of the Heat Equation	390
22. A Solution for the Joule-Heat Source Term	391
22.1 Physical Introduciton	391
22.2 Form of the Source Function and Particular Solution	393
22.3 The Joule Heating Spectrum	395
22.4 The Range of Values $\alpha_1, \alpha_2, \alpha_3, \sigma$ and Asymptotic Solution of τ -Integral	399
22.5 Evolution of τ -Integral Eq. (22.29)	401
23. Results for the $\overline{\theta^2}$ Spectrum with Joule Heating	403
23.1 The Asymptotic Behavior of the Solutions	403
23.2 The Most Probable Form of the $\overline{\theta^2}$ -Spectrum	417
Chapter V. Contemporary Turbulence	423
24. Recent Developments in Turbulence Through Use of Experimental Mathematics - Attractor Theory	423
24.1 Things That Change Suddenly	423
24.2 Order in the Chaos	424
24.3 Attractor Theory in Turbulent Channel Flows	430
25. Recent Developments in Experimental Turbulence	447
25.1 Coherent Structure of Turbulent Shear Flows	448
Appendices	451
Appendix A -- Derivation of Correlation Equations (13.51-13.62)	451
Appendix B -- Derivation of Spectrum Equations (14.45-14.46)	460
Appendix C -- Fourier Transforms (18.10)	471
Appendix D -- The Time Variation of Eq. (18.3)	475
Appendix E -- The Time Variation of Eq. (18.19)	478
Bibliography	482
Author Index	490
Subject Index	495